where

$$G = k_{1y}^2\left(k_{1x}^2 - \frac{B}{2}\right) - \left(\frac{m\pi}{a}\right)^2 k_{1x}^2$$

$$J = -\ 2Fk_{1y}^2$$

$$\phi(u, v, w) = 2F\mu[j\gamma X(ur + vq) + Kk_{1x}^2 w].$$

The unknowns here are the three magnetic field amplitudes and r. The electric amplitudes are known in terms of these parameters. The theory leading to these equations has been rigorous and they are now ready for solution by numerical methods or by approximation techniques.

It can be shown that there are no pure TE or pure TM modes allowed in the magnetized case. A similar result was found by Gamo and Kales in their treatment of the longitudinally magnetized cylindrical waveguide. This is physically reasonable since the transverse magnetic fields for the TM modes now generate longitudinal fields through the rotational nature of the ferrite and thus TM modes would not be expected. Maxwell's equations permite TE modes only for modes with zero x dependence and these are Van Trier's TE_{no} modes.

In conclusion we have derived a set of four nonlinear equations whose solution determines a rigorous solution to the problem of propagation in a transversely magnetized ferrite-filled waveguide. The fields can be expressed in the form of products of two trigonometric functions with arguments which are asymptotic to $n\pi y/b$ and 0 in the limit of zero applied field. The product of these arguments is dependent only on the magnetic field and frequency.

Currents Excited on a Conducting Surface of Large Radius of Curvature

JAMES R. WAIT†

Summary—**The nature of the electromagnetic field of an antenna in the vicinity of a surface of large radius of curvature is discussed. Assuming a spherical surface, the solution for a dipole source in the form of the Watson residue series is transformed to a more rapidly converging series which is preferable at short distances. Using this result, numerical data is presented in graphical form for the currents induced on the spherical surface. The curves are applicable to both a stub and slot antenna mounted on the conducting surface.**

IN THE VICINITY of a flush-mounted radar antenna for aircraft, the fuselage is a smooth conducting surface having large radii of curvature. It is of interest to know how the current distribution excited on this curved surface differs from that on a perfectly flat surface. It is the purpose of this paper to investigate this problem by simulating the curved surface in the vicinity of the antenna by a spherical surface.

The starting point is to consider the fields of a radial electric dipole located on a perfectly conducting sphere of radius a. Choosing a spherical coordinate system (r, θ, ϕ), the dipole is located at $r=a$ on the polar axis and the sphere is bounded by $r=a$. As is well known,[1] the solution of this problem can be expressed in a radial mode series involving half-order Bessel function whose arguments are ka where $k=2\pi$/wavelength. Unfortunately, this representation which is often called the *harmonic series* is very poorly convergent if ka is large compared to unity. In fact, something of the order of 2 ka terms are required to evaluate the field at any point in space. It was first shown by Watson in 1918 that the radial mode or harmonic series could be transformed to the angular mode or residues series.[1] This Watson representation is highly convergent in certain regions of space, namely, deep in the geometrical shadow of the source. However, when the observer is in the space near the dipole source, the series becomes poorly convergent.

It is the principal task in this paper to derive an alternative expansion which is particularly suitable for calculating the surface currents excited on the spherical surface when θ is small and ka is large.

The fields of the dipole can be expressed in terms of a scalar function v as follows:[1]

$$E_r = \left(k^2 + \frac{\partial^2}{\partial r^2}\right)(rv)$$

$$E_\theta = \frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}(rv)$$

$$H_\phi = -\ i\epsilon\omega\frac{\partial v}{\partial\theta} \quad (1)$$

where $\epsilon=8.854\times10^{-12}$ and the time factor exp $(i\omega t)$ has been omitted. The surface current density I, in amperes/meter, on the spherical surface has only a radial component. It is given by

$$I = H_\phi\big]_{r=a}$$

† Natl. Bur. of Standards, Boulder, Colo.

[1] H. Bremmer, "Terrestial Radio Waves," Elsevier Pub. Co. Amsterdam, Holland; 1949.

Using the residues-series representation for v developed by Van der Pol and Bremmer[1] it is easy to show that

$$I = I_0 U \tag{2}$$

where I_0 is the current density for a sphere of infinite radius or a flat surface and

$$U = [2\pi i x^{2/3}]^{1/2} \sum_{q=0}^{\infty} \frac{\exp[-i\tau_q x^{2/3}]}{2\tau_q} \tag{3}$$

where $x=(kd)^{3/2}/ka$ with $d=a\theta$. The coefficient τ_q is the qth root of

$$H_{2/3}^{(2)}[1/3(-2\tau_q)^{3/2}] = 0 \tag{4}$$

where $H_{2/3}^{(2)}[z]$ is the Hankel function of the second kind of order $\frac{2}{3}$ and argument z. The roots of the Hankel function are tabulated by Watson.[2] Eq. (3) is an approximation valid for a, large compared to both the wavelength and the distance d, from the source to the observer.

Following a crucial suggestion of Dr. H. Bremmer[3] U is now expressed as a Bromwich contour integral as follows

$$U = \frac{1}{2\pi i} g^{1/2} e^{-5\pi/6} \cdot \int_{c-i\infty}^{c+i\infty} \frac{e^{sg}}{s^{1/2}} \frac{H_{1/3}^{(2)}\left[\frac{(-2i)^{3/2}}{3} s^{3/2}\right]}{H_{2/3}^{(2)}\left[\frac{(-2i)^{3/2}}{3} s^{3/2}\right]} ds \tag{5}$$

where $g=x^{2/3}$ and c is some positive real constant. It can be verified by the theory of functions that the sum of the residues at the poles of the integrand leads back to the series representation for U in (3). Symbolically the above equation can be written

$$LU = \frac{g^{1/2} e^{-i5\pi/6}}{s^{1/2}} \frac{H_{1/3}^{(2)}\left[\frac{(-2i)^{3/2}}{3} s^{3/2}\right]}{H_{2/3}^{(2)}\left[\frac{(-2i)^{3/2}}{3} s^{3/2}\right]} \tag{6}$$

where the operator L indicates that the right-hand side of this equation is the Laplace transform[4] with respect to g of U.

It is now proposed to obtain a series formula for U in positive powers of g by developing LU in an asymptotic expansion in power of $1/s$. In order to develop the Hankel functions[2] in their asympotic expansions, it is necessary to assure that the phase of the argument lies within the range -2π to π. Noting that s will range in values from $+i\infty$ to $-i\infty$ it is desirable to multiply the arguments of the Hankel functions in (6) by $e^{-3i\pi}$.

Using the identity[2]

$$\frac{H_{1/3}^{(2)}[z]}{H_{2/3}^{(2)}[z]} = e^{i\pi} \frac{H_{1/3}^{(2)}[ze^{-3i\pi}]}{H_{2/3}^{(2)}[ze^{-3i\pi}]} \tag{7}$$

it follows that

$$LU = (g/s)^{1/2} e^{i\pi/6} \frac{H_{1/3}^{(2)}[-i^{1/2}(2s)^{3/2}/3]}{H_{2/3}^{(2)}[-i^{1/2}(2s)^{3/2}/3]}. \tag{8}$$

Denoting $-i^{1/2}(2s)^{3/2}/3$ by Z, the following asymptotic expansions[2] are now valid:

$$H_{1/3}^{(2)}(Z) = \left(\frac{2}{\pi Z}\right)^{1/2} e^{-i(Z-(5\pi/12))} \sum_{m=0}^{\infty} \frac{(-1)^m(1/3, m)}{(2iZ)^m} \tag{9}$$

and

$$H_{2/3}^{(2)}(Z) = \left(\frac{2}{\pi Z}\right)^{1/2} e^{-i(Z-(7\pi/12))} \sum_{m=0}^{\infty} \frac{(-1)^m(2/3, m)}{(2iZ)} \tag{10}$$

with

$$(\nu, m) = \frac{(\nu + m - 1/2)!}{m!(\nu - m - 1/2)!}.$$

It then follows that

$$\frac{H_{1/3}^{(2)}(Z)}{H_{2/3}^{(2)}(Z)} = e^{-i\pi/6}\left[1 + \frac{i}{6Z} - \frac{7}{72Z^2} - i\frac{7}{72Z^3} + \cdots\right]. \tag{11}$$

The desired asymptotic expansion is then given by

$$LU = (\pi g)^{1/2}\left[\frac{1}{s^{1/2}} - \left(\frac{i}{2}\right)^{1/2} \frac{1}{4s^2} + i\frac{7}{64s^{7/2}} + \left(\frac{1}{2i}\right)^{1/2} \frac{21}{128s^5} + \cdots\right]. \tag{12}$$

Employing the basic relation[3]

$$\frac{1}{s^\nu} = \int_0^\infty \frac{g^{\nu-1}}{(\nu-1)!} e^{-gs} dg = L\frac{g^{\nu-1}}{(\nu-1)!} \tag{13}$$

it now readily follows that

$$U = 1 - \frac{\pi^{1/2}}{8}(1+i)g^{3/2} + \frac{i7}{120}g^3 + \frac{\pi^{1/2}7(1-i)}{2048} g^{9/2} + \cdots \tag{14}$$

or

$$U = \left[1 - \frac{\pi^{1/2}}{8}x + \frac{7\pi^{1/2}}{2048}x^3 + \cdots\right] - i\left[\frac{\pi^{1/2}}{8}x - \frac{7}{120}x^2 + \frac{7\pi^{1/2}}{2048}x^3 + \cdots\right] \tag{15}$$

where $x=(kd)^{3/2}/ka$.

[2] G. N. Watson, "Theory of Bessel Functions," 2nd ed., 1945.
[3] Private communication.
[4] R. V. Churchill, "Operational Methods in Engineering," McGraw-Hill Book Co. Inc., New York, N.Y.; 1944.

It can be seen that as d tends to zero or a tends to infinity, the value of U approaches unity corresponding to the dipole on a flat conducting surface.[5]

The series, which can be called the 3rd order curvature corrected series, converges adequately for x less than about 1.5. Writing

$$U = |U| e^{-i\Phi}$$

then $|U|$ and Φ are the amplitude and phase lag, respectively, of the correction factor U. Numerical values are computed from (15) and shown plotted in Fig. 1 for x ranging from 0.1 to 3. Values of $|U|$ and Φ obtained from the residue series formula in (3) are also shown on Fig. 1. The agreement between the two sets of curves is excellent unless x exceeds about 2.0. When x is of the order of 0.1 or less, greater than 100 terms in the residue series are required to obtain three figure accuracy, whereas only the term in x need be retained in the curvature corrected series for U. At larger values of x, say greater than 2 or 3, only several terms in the residue series are required, whereas the curvature corrected series would be very poorly convergent.

Although the preceding theory was developed explicitly for a radial electric dipole source, the results are directly applicable to the current excited on a spherical surface by a narrow slot or its equivalent magnetic dipole. Eq. (2) relating the current I on the curved surface to the current I_0 on a flat surface is only strictly valid in the broadside direction from the narrow slot as

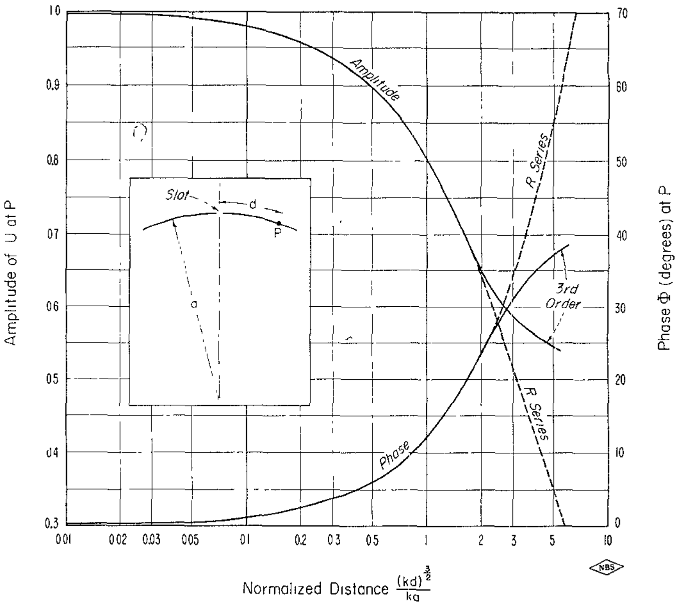

Fig. 1—The ratio of the current induced by a slot on a curved surface to that on a flat surface.

indicated in the inset in Fig. 1. If kd is much greater than unity, however, the equation is also valid in other directions from the slot. It can now be expected that the mutual impedance Z_m between any two slots oriented for other than minimum coupling on a spherical surface of large radius of curvature a is related to the mutual impedance Z_{0m} for the same slots on a flat surface by the formula

$$Z_m \cong U Z_{0m}.$$

In this case, d is taken as the distance between the centers of the slots.

[5] Bremmer, *loc. cit.*, has developed expansion formulas, similar to (15), which are expressed in powers of a factor δ which is approximately proportional to the complex refractive index of the sphere, which in the present analysis is effectively infinite.

A Note on Noise Temperature

PETER D. STRUM†

Summary—**The effective noise temperature of the output impedance of a lossy passive network at an arbitrary noise temperature connected to one or more resistive loads at arbitrary noise temperature lies between the highest and the lowest of these noise temperatures, as determined by the losses between the output terminals and the loads. The determination of the effective noise temperature of a gas-discharge noise generator over a wide frequency range is simplified by the substitution of a loss measurement for the more difficult noise temperature measurement. For minimum-noise radar applications care must be used in considering the excess noise of crystal mixers and gas-discharge duplexers. The influence of galactic radiation on a receiving system is such that there is an optimum frequency in the region of 200 to 600 mc for minimum "operating noise figure." Typical examples of radio-astronomy measurements are amenable to analysis of the type given. Finally, several corrections to measured noise figure are analyzed.**

† Ewen Knight Corp., Needham Heights, Mass. Formerly at Airborne Instruments Lab., Mineola, N.Y.

Introduction

The OUTPUT noise power of many widely used devices is conveniently expressed in terms of an equivalent noise temperature—that is, the temperature of a passive resistor that would generate an equivalent available noise power. In the case of directional antennas and gas-discharge noise generators, the term noise temperature is accurately applied because, in one case, the antenna is directed into space, which